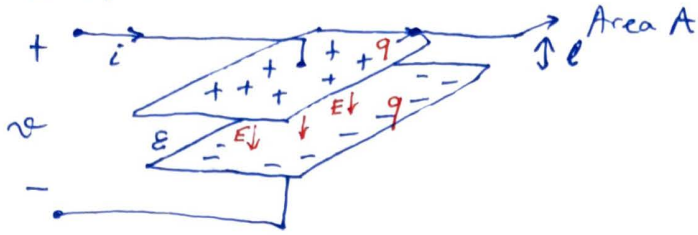
ENERGY STORAGE ELEMENTS

Time has not been taken into account so far. Capacitive and inductive effects change this.

Capacitor



The charge accumulates on plates. (+, − charge on each plates). There is electric field between plates

$$E(t) = \frac{q(t)}{\varepsilon A(t)}, \quad v(t) = \ell(t) \cdot E(t)$$

ε: dielectric permitivity for insulator. so $q(t) = \underbrace{\frac{\varepsilon A(t)}{\ell(t)}}_{C(t)} \cdot v(t) = C(t)\, v(t)$

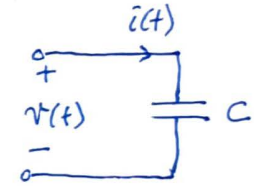
C: capacitance (Coulombs / Volt : Farad (F))

Resistor exhibits proportional relation between current and voltage.
capacitor exhibits proportional relation between voltage and stored charge.
Rate of charge transfer is current: $\frac{dq(t)}{dt} = i(t)$ (Coulomb/second : Ampere)

C(t) capacitance is usually time invariant: C

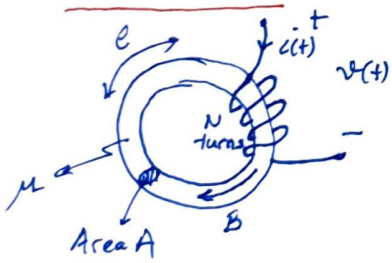
$$q(t) = C\, v(t)$$

$$\boxed{i(t) = C \frac{d}{dt} v(t)} \quad \text{or} \quad v(t) = \frac{1}{C} \int_{-\infty}^{t} i(t)\, dt .$$



* Stores electric field!

Inductor



$$B(t) = \frac{\mu N\, i(t)}{\ell(t)}, \quad \Phi(t) = A(t) \cdot B(t)$$

B(t): density of flux; $\Phi(t)$: magnetic flux

$$\lambda(t) = N \Phi(t) = \underbrace{\frac{\mu N^2 A(t)}{\ell(t)}}_{L(t)}\, i(t)$$

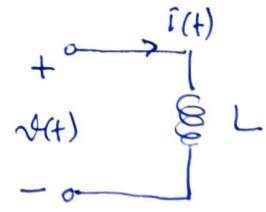
$\lambda(t)$: total flux

L: inductance (webers / Ampere : Henry (H))

L(t) is usually time invariant: L. $\frac{d\lambda(t)}{dt} = v(t)$

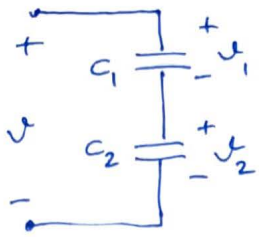
$$\boxed{v(t) = L \frac{di(t)}{dt}} \quad \text{or} \quad i(t) = \frac{1}{L} \int_{-\infty}^{t} v(t)\, dt$$

* Stores magnetic field!



ENERGY STORAGE ELEMENTS

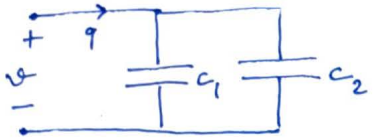
Series and Parallel Connections



$$q(t) = C_1 v_1(t) + C_2 v_2(t)$$

$$v(t) = v_1(t) + v_2(t)$$

$$\frac{1}{C} = \frac{v(t)}{q(t)} = \frac{1}{C_1} + \frac{1}{C_2}, \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

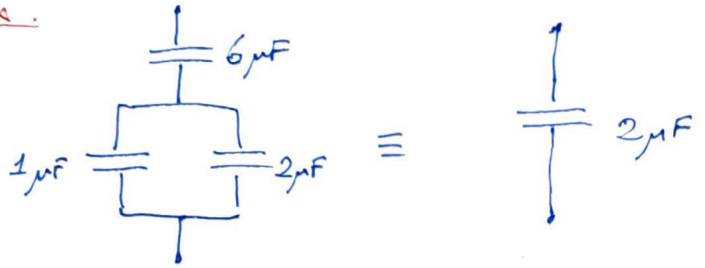


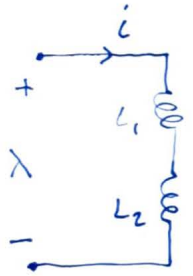
$$v(t) = \frac{q_1(t)}{C_1} = \frac{q_2(t)}{C_2}$$

$$q(t) = q_1(t) + q_2(t)$$

$$C = \frac{q(t)}{v(t)} = C_1 + C_2$$

Ex.

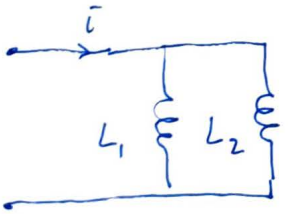




$$i(t) = \frac{\lambda_1(t)}{L_1} = \frac{\lambda_2(t)}{L_2}$$

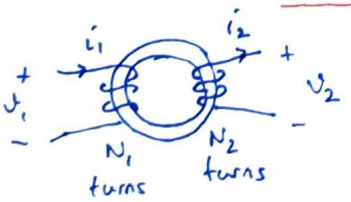
$$\lambda(t) = \lambda_1(t) + \lambda_2(t)$$

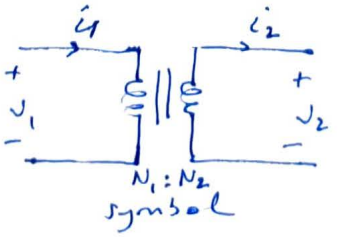
$$L = \frac{\lambda(t)}{i(t)} = L_1 + L_2$$



Similarly, $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L = \frac{L_1 \cdot L_2}{L_1 + L_2}$

TRANSFORMERS





$$v_1 = N_1 \frac{d\phi(t)}{dt}$$

$$v_2 = N_2 \frac{d\phi(t)}{dt}$$

$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$

$$N_1 i_1(t) = N_2 i_2(t)$$