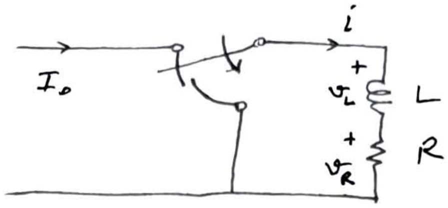
FIRST-ORDER CIRCUITS/SYSTEMS

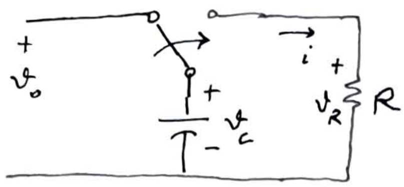
The highest degree of derivative is 1. → RC, LR networks:

L-R example:



KVL: $\sum v = 0 = -v_L - v_R = -L\frac{di}{dt} - Ri$

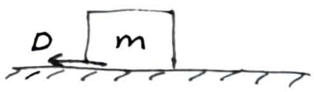
$$L\frac{di}{dt} + Ri = 0$$



KVL: $\sum v = 0 = v_C - v_R = v_0 - \frac{1}{C}\int_0^t i\,dt - Ri$

$$R\frac{di}{dt} + \frac{1}{C}i = 0$$

Analogy with mechanics:



→ v (v_0 @ $t=0$)

D: coefficient of friction. Driving force is removed at $t=0$. At $t=0$, $v = v_0$.

$$\sum f = 0 = -m\frac{dv}{dt} - Dv$$

$$m\frac{dv}{dt} + Dv = 0$$

Solving the Equation:

$R\frac{di}{dt} + \frac{1}{C}i = 0$ says that combination of $i(t)$ and $\frac{d}{dt}i(t)$ must equal zero.

This is possible if the function is exponential.

$i = Ae^{st} \Rightarrow \frac{di}{dt} = sAe^{st}$ $\qquad RsAe^{st} + \frac{1}{C}Ae^{st} = 0$

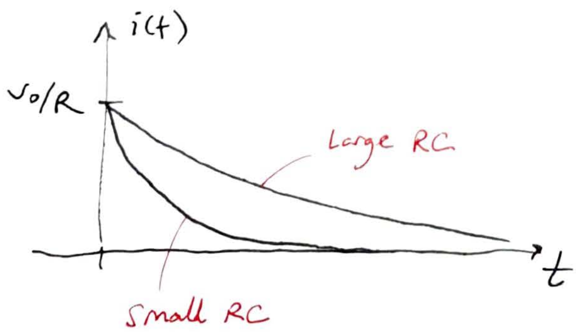
$$\underbrace{\left(Rs + \frac{1}{C}\right)}_{0} \underbrace{Ae^{st}}_{\neq 0} = 0$$

$s = -\frac{1}{RC}$ $\qquad$ Therefore $\quad i(t) = Ae^{(-1/RC)t}$. what is A?

we return to the original equation:

Initially $v_C = v_0$ at time $t=0$. Therefore $i(0) = \frac{v_C}{R} = \frac{v_0}{R}$

$i(0) = Ae^{-1/RC \cdot 0} = A = \frac{v_0}{R} \quad \Rightarrow \quad i(t) = \frac{v_0}{R}e^{-\frac{t}{RC}}$

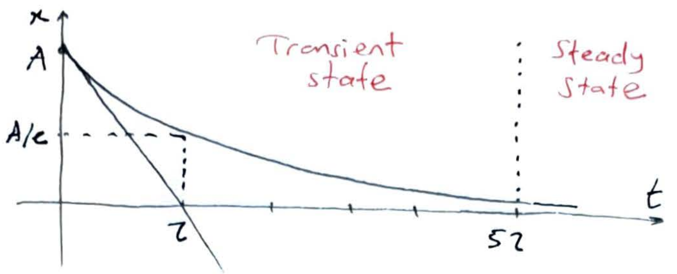


$s + \frac{1}{RC} = 0$ is said to be the characteristic equation of the system.

The product $R \cdot C$ is called <u>time constant</u>. $(\tau = RC)$

Some features of Ae^{st} :

$x = Ae^{-t/\tau}$. The initial slope of x is $\left.\frac{dx}{dt}\right|_{t=0} = -\frac{A}{\tau}$

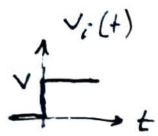


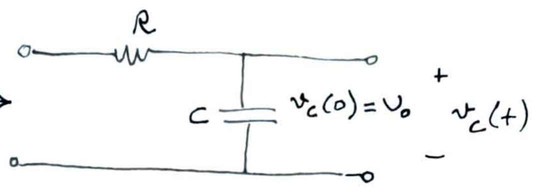
$x(0) = A$

$x(\tau) = \frac{A}{e}$

Since $e^{-5} = 0.0067$ is very small
It's common to assume that for $t > 5\tau$ the function is zero. So system is said to have completed transient response in 5τ time unit

Step response :



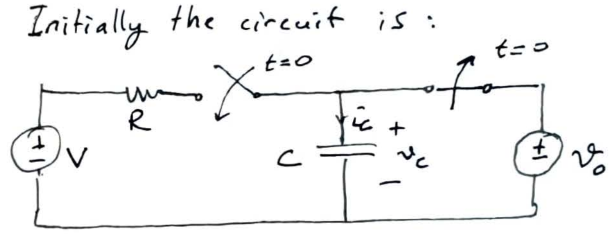


KCL : $\frac{v_c - v_i}{R} + C\frac{dv_c}{dt} = 0$

$v_i = V \cdot u(t)$

$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

Initially the circuit is :



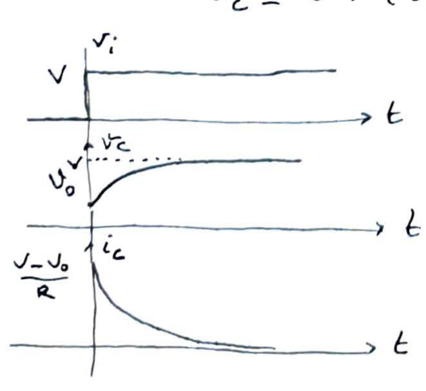
$v_c(0) = v_0$

$v_c(\infty) = V$

So we can write : $v_c = V + Ae^{-t/RC}$, $V_c(0) = v_0 \Rightarrow V + A = v_0$

$V_c(\infty) = V \Rightarrow A = v_0 - V$,

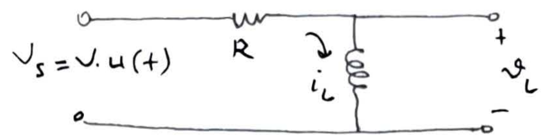
$v_c = V + (v_0 - V)e^{-t/RC}$



$$v_c = \underbrace{V}_{\text{steady state output}} + \underbrace{(v_0 - V)e^{-t/RC}}_{\text{transient output}}$$

$$i_c = C\frac{dv_c}{dt} = \frac{V - V_0}{R}e^{-t/RC}$$

Analysis of R-L circuit



$$-v_s + i_L \cdot R + L\frac{di_L}{dt} = 0$$

$$-\frac{V}{s} + I_L(s)\cdot R + L\left(s\,I_L(s) - \overset{0}{i_L(0)}\right) = 0$$

$$I_L \cdot R + Ls\,I_L = V/s$$

$$s.\,I_L(R+sL) = V \Rightarrow I_L = \frac{V}{s(R+sL)}$$

Laplace Trasforms

$f(t)$	$F(s)$
$A\,u(t)$	A/s
$f'(t)$	$sF(s) - f(0)$
e^{-at}	$\frac{1}{s+a}$
$f''(t)$	$s^2F(s) - s\,f(0) - f'(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$a\,t\,u(t)$	A/s^2

$$I_L(s) = \frac{V}{s(R+sL)} = \frac{A}{\underset{(R+sL)}{s}} + \frac{B}{\underset{s}{R+sL}}$$

$$\frac{AR + AsL + Bs}{s(R+sL)} = \frac{V}{s(R+sL)} \Rightarrow$$

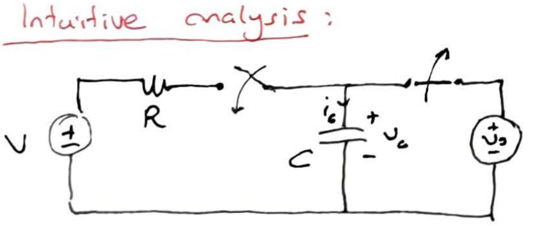
$$s\underbrace{(AL+B)}_{0} + \underbrace{AR}_{V} = V \Rightarrow A = \frac{V}{R} \quad B = -\frac{VL}{R}$$

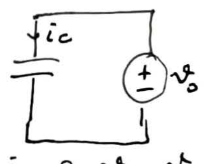
$$I_L(s) = \frac{V/R}{s} - \frac{VL/R}{R+sL} = \frac{V/R}{s} - \frac{V/R}{\frac{R}{L}+s}$$

inv. Laplace trasform : $i_L(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$

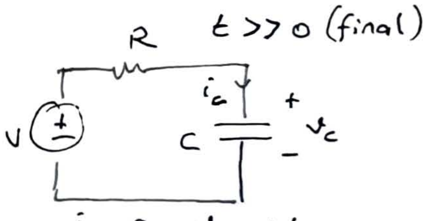
$$v_L = L.\frac{di_L}{dt} = \cancel{L}\left(-\frac{V}{\cancel{R}}\right).-\frac{\cancel{R}}{\cancel{L}}e^{-R/L\,t} = V e^{-R/L\,t}$$

Intuitive analysis :

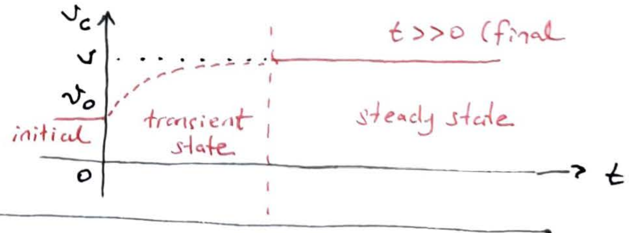




$i_c = 0,\ v_c = v_o$



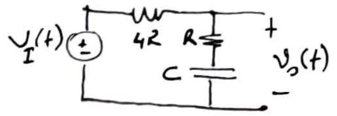
$i_c = 0,\ v_c = V$



$$\boxed{v_c = \text{initial value } e^{-t/\text{time constat}} + \text{final value}\left(1 - e^{-t/\text{time constant}}\right)}$$

$$v_c = v_o e^{-t/RC} + V\left(1 - e^{-t/RC}\right) = \underbrace{V}_{\text{steady state component}} + \underbrace{(v_o - V)\,e^{-t/RC}}_{\text{trasient component}}$$

Exercise 10.18 (pg. 572)



Find the characteristic equation. Assume C is initially charged to 1 volt. Plot $v_o(t)$.