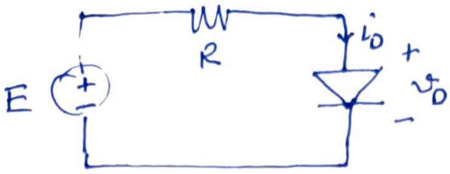
NONLINEAR CIRCUITS

Solve analytically non-linear circuit with diode:



$$\frac{v_D - E}{R} + i_D = 0$$

$$i_D = I_S\left(e^{v_D/V_{TH}} - 1\right)$$

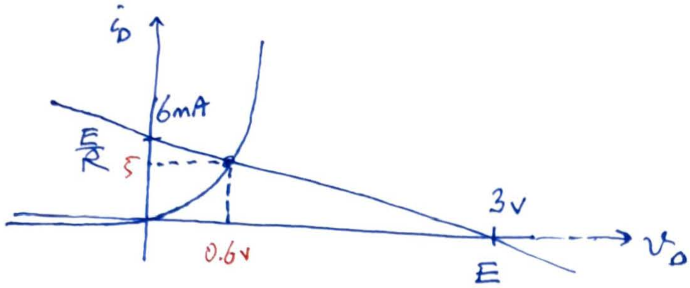
$$\Rightarrow \frac{v_D - E}{R} + I_S\left(e^{v_D/V_{TH}} - 1\right) = 0$$

not easy to solve!
numerical solution seems easier.

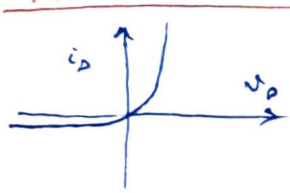
Graphical analysis:

$E = 3v$, $R = 500\Omega$, find i_D, v_D ?

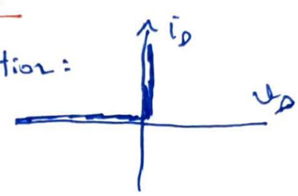
$$i_D = \frac{E - v_D}{R}$$



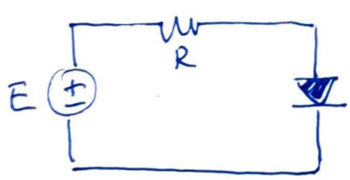
Piecewise Linear Analysis



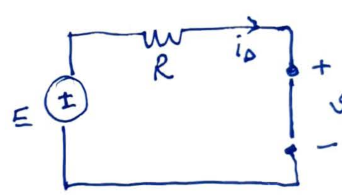
$\Rightarrow$ Assumption:



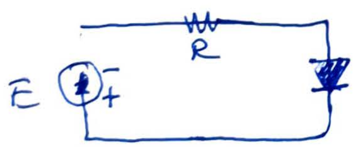
Diode ON (short circuit):
$v_D = 0$ for all $i_D > 0$
Diode OFF (opencircuit):
$i_D = 0$ for all $v_D < 0$.



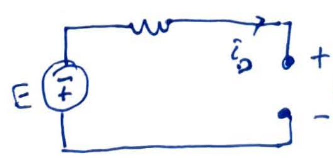
$\Rightarrow$



Short circuit ($v_D = 0$, $i_D = \frac{E}{R}$)

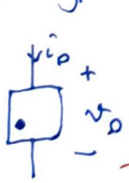


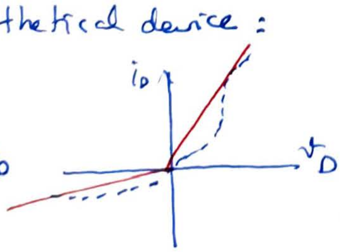
$\Rightarrow$



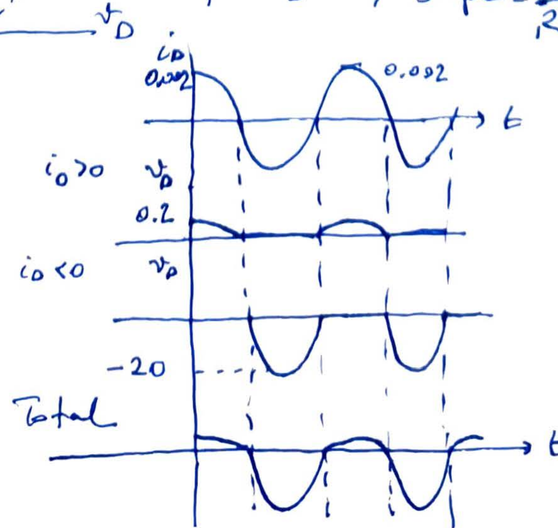
open circuit ($v_D = E$, $i_D = 0$)

A hypothetical device:



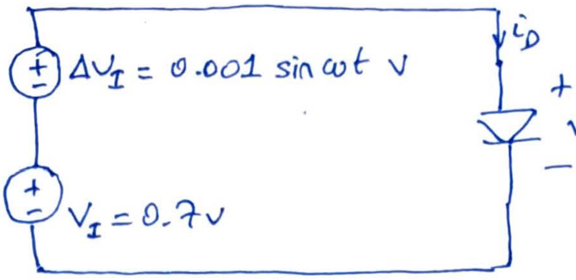


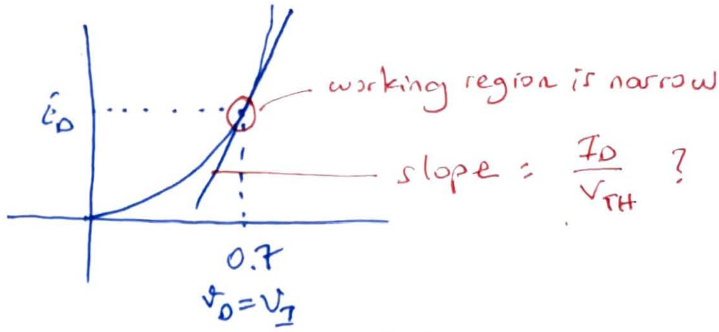
for $i_D < 0$, slope $= \frac{1}{R_2}$ ($R_2 = 10k\Omega$)
for $i_D > 0$, slope $= \frac{1}{R_1}$ ($R_1 = 100\Omega$)



NON-LINEAR CIRCUITS

Incremental Analysis :





Taylor exp. at V_D :

$$i_D = I_s\left(e^{(0.7 + 0.001 \sin \omega t)/V_{TH}} - 1\right)$$

$$i_D = F(v_D) = f(V_D) + \frac{df}{dv_D}\Big|_{V_D}(v_D - V_D) + \frac{1}{2!}\frac{d^2 f}{dv_D^2}\Big|_{V_D}(v_D - V_D)^2 + \dots$$

We wish to expand diode equation $i_D = I_s\left(e^{(V_D + \Delta v_D)/V_{TH}} - 1\right)$ about the point V_D, I_D. When we apply $v_D = V_D + \Delta v_D$, the current will be

$$i_D = I_D + \Delta i_D .$$

Taylor expansion :

$$i_D = I_s\left(e^{V_D/V_{TH}} - 1\right) + \frac{1}{V_{TH}}\left(I_s e^{V_D/V_{TH}}\right)\Delta v_D + \frac{1}{2}\left(\frac{1}{V_{TH}}\right)^2 I_s e^{V_D/V_{TH}} (\Delta v_D)^2 + \dots$$

Simplifying :
$$i_D = I_s\left(e^{V_D/V_{TH}} - 1\right) + \left(I_s e^{V_D/V_{TH}}\right)\left[\frac{1}{V_{TH}}\Delta v_D + \frac{1}{2}\left(\frac{1}{V_{TH}}\right)^2 (\Delta v_D)^2 + \dots\right]$$

$\underset{0.001}{\Delta v_D}$ is very small compared to $\underset{0.025}{V_{TH}}$, so we can ignore higher order terms :

$$i_D = \underbrace{I_s\left(e^{V_D/V_{TH}} - 1\right)}_{I_D} + \underbrace{I_s e^{V_D/V_{TH}} \cdot \frac{1}{V_{TH}} \cdot \Delta v_D}_{\Delta i_D}$$

This is $I_D + \Delta i_D \Rightarrow$

$$\Delta i_D = \underbrace{\left(I_s e^{V_D/V_{TH}}\right)}_{I_D} \frac{1}{V_{TH}} \cdot \Delta v_D \quad \text{(Linear equation)}$$

$$\Delta i_D = I_D \cdot \frac{1}{V_{TH}} \cdot \Delta v_D \Rightarrow \text{slope of the line} : \frac{I_D}{V_{TH}}$$

For more accurate calculation we can take higher terms of Taylor.

$$v_D = V_D + \Delta v_D = 0.7\text{V} + 0.001\text{V} \cdot \sin \omega t$$

so Δv_D is small enough : $i_D = I_D + \Delta i_D \qquad I_D = I_s\left(e^{0.7/V_{TH}} - 1\right)$

$$\Delta i_D = \frac{df}{dv_D}\Big|_{V_D} \Delta v_D = I_s e^{0.7/V_{TH}} \frac{1}{V_{TH}} \cdot 0.001 \sin \omega t$$

$$I_D = 1.45\text{ A}, \quad \Delta i_D = 0.058\text{ A} \cdot \sin \omega t .$$