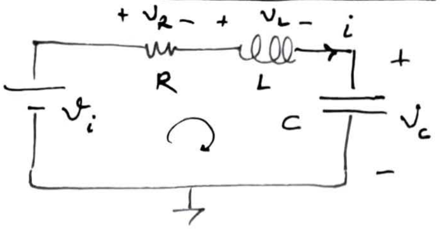
Driven series R-L-C :



$i = C\frac{d}{dt}v_c(t)$ or $i = C v_c'$

$v_L = L \cdot i'$ => $v_L = LC v_c''$

KVL : $-v_i + Ri + LCv_c'' + v_c = 0$

$$v_c'' + \frac{R}{L}v_c' + \frac{1}{LC}v_c = \frac{1}{LC}v_i$$

or $$\frac{d^2}{dt}v_c(t) + \frac{R}{L}\frac{dv_c}{dt}(t) + \frac{1}{LC}v_c(t) = \frac{1}{LC}v_i(t)$$

$v_i(t) = A.u(t)$. Apply Laplace tr. : $s^2V_c + \frac{R}{L}sV_c + \frac{1}{LC}V_c = \frac{1}{LC}\cdot\frac{A}{s}$

$\Rightarrow V(s) = \dfrac{\frac{A}{LCs}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$ $\Rightarrow$ $v_c(t) = ?$ (Find using inv. Laplace tr.)

State Equations :

$-v_i + Ri + L\frac{di}{dt} + v_c = 0$, $i = C\frac{dv_c}{dt}$

$\frac{di}{dt} = -\frac{R}{L}i - \frac{v_c}{L} + \frac{v_i}{L}$ $\qquad$ $\frac{dv_c}{dt} = \frac{i}{C}$

$i^+ = i - \frac{R}{L}i.\Delta t - \frac{v_c}{L}\Delta t + \frac{v_i}{L}\Delta t$ $\qquad$ $v_c^+ = v_c + \frac{i}{C}\Delta t$ $\qquad$ $\boxed{\text{rlc.py}}$ *

$v_c(t) = K_1e^{s_1t} + K_2e^{s_2t}$, $s^2 + 2\alpha s + \omega_0^2 = 0$ where $\alpha \equiv \frac{R}{2L}$ $\quad \omega_0 \equiv \frac{1}{\sqrt{LC}}$

the roots are $s_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2}$

for $\alpha < \omega_0$ s_{1-2} are complex : $s_1 = -\alpha + j\omega_d$ where $\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$
$s_2 = -\alpha - j\omega_d$

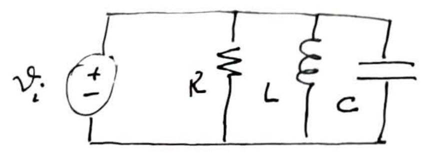
Both oscillatory and decaying behavior : $v_c(t) = A_1e^{-\alpha t}\cos\omega_d t + A_2e^{-\alpha t}\sin\omega_d t$

$v_c(0) = V_0 + A_1$ $\qquad$ ($V_{in}(t) = V_0\, u(t)$) step input
$i_L(0) = \alpha CA_1 - \omega_d CA_2$

pg 658 : $A_1 = -V_0$
$A_2 = -\frac{\alpha}{\omega_d}V_0$ $\Rightarrow$ $V_c(t) = V_0\left(1 - \frac{\omega_0}{\omega_d}e^{-\alpha t}\cos\left(\omega_d t - \tan^{-1}\frac{\alpha}{\omega_d}\right)\right)u(t)$

$i_L(t) = -\frac{V_0}{\omega_d L}e^{-\alpha t}\sin\omega_d t\; u(t)$

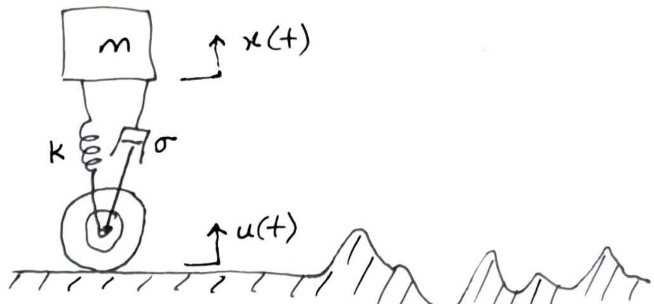
* write python code to simulate driven parallel R-L-C circuit:
Plot i_R, i_L, i_c . $v_i(t) = 10\,u(t)$
$= 10\sin\omega t$
$= 10\,t\,u(t)$
⋮



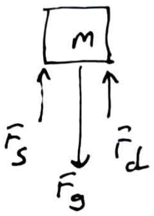
HIGHER ORDER CIRCUITS / SYSTEMS

Analogy with mechanical systems:

Electrical	Mechanical
current (i)	force (f)
voltage (v)	velocity (v)
resistance (r)	lubricity ($1/B$) (inv. friction)
capacitance (c)	mass (m)
inductance (l)	compliance ($1/k$) (inv. spring)

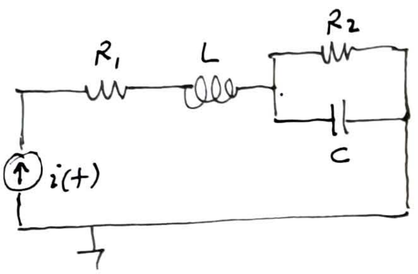


$\Sigma f = 0$

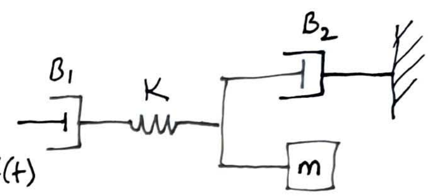


$$F_g = mg$$
$$F_s = k(x-u)$$
$$F_d = \sigma(\dot{x}-\dot{u})$$

$$\Rightarrow \boxed{m\ddot{x} + \sigma\dot{x} + kx = ku + \sigma\dot{u}}$$



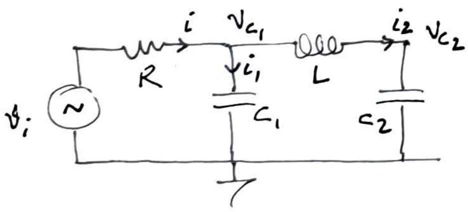
$\rightsquigarrow$



* Find the resonant frequency for undriven LC, or spring-mass:

we know that $\omega_0 = \frac{1}{\sqrt{LC}} \rightsquigarrow \omega_0 = \sqrt{\frac{k}{m}}$

Third-order circuit (Low-pass filter application)



$i = i_1 + i_2$

$i_1 = C_1 \frac{d}{dt} v_{c_1}$

$i_2 = C_2 \frac{d}{dt} v_{c_2}$

$v_i - v_{c_1} = R(i_1 + i_2) \Rightarrow i_1 = \frac{v_i}{R} - \frac{v_{c_1}}{R} - i_2$ *

$v_{c_1}' = \frac{i_1}{C_1}$ * , $v_{c_2}' = \frac{i_2}{C_2}$ *

$L i_2' = v_{c_1} - v_{c_2} \Rightarrow i_2' = \frac{1}{L}(v_{c_1} - v_{c_2})$ *

- write state equations
- simulate circuit with $\boxed{\texttt{rc1lc2.py}}$

for v_i:
1) step input
2) ramp input
3) $\sin(\omega t)$ for different ω values.
4) a noisy $\sin(\omega t)$ to observe filter effect.

Recall: $\boxed{\omega = 2\pi f}$

play with l, c values to see cut-off frequency and phase difference!