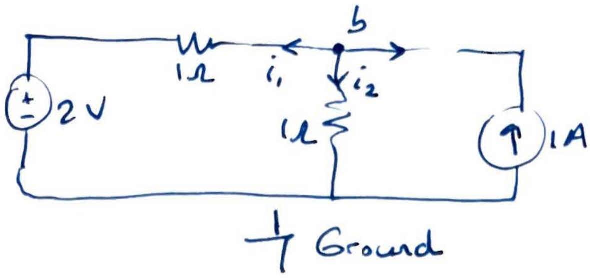
NETWORK THEOREMS

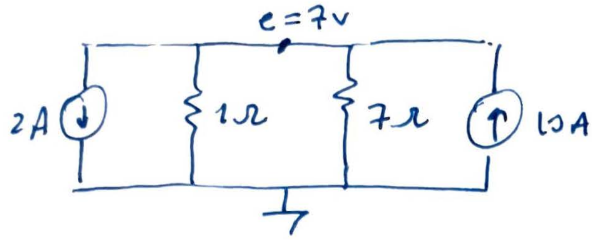
Node Voltage



$$\frac{v_b}{1} + \frac{v_b - 2}{1} - \frac{1}{1} = 0 \Rightarrow v_b = 1.5\,V$$

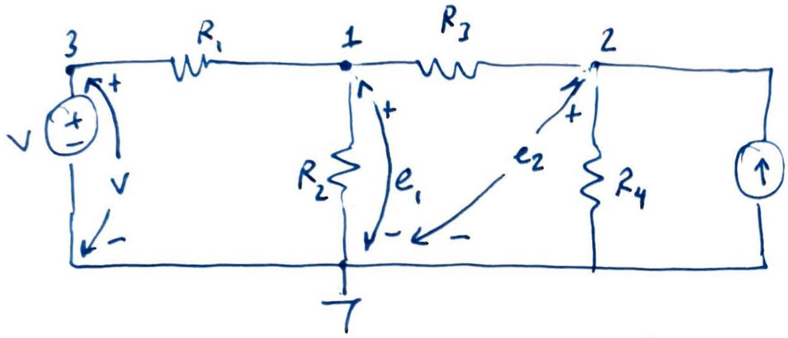
$$i_1 = \frac{1.5 - 2}{1} = -0.5\,A$$

$$i_2 = \frac{1.5}{1} = 1.5\,A$$



show node voltage $e = 7V$ satisfies KCL.

$$2A + \frac{(7-0)V}{1\Omega} + \frac{(7-0)V}{7\Omega} - 10A = \emptyset$$



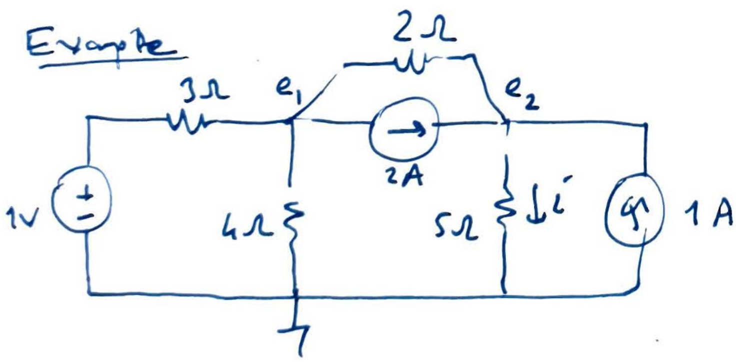
KCL: node 1: $\frac{v - e_1}{R_1} + \frac{e_2 - e_1}{R_3} - \frac{e_1}{R_2} = 0$

node 2: $\frac{e_1 - e_2}{R_3} - \frac{e_2}{R_4} + I = 0$

$$\frac{v}{R_1} = e_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{e_2}{R_3}$$

$$I = -\frac{e_1}{R_3} + e\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

Example



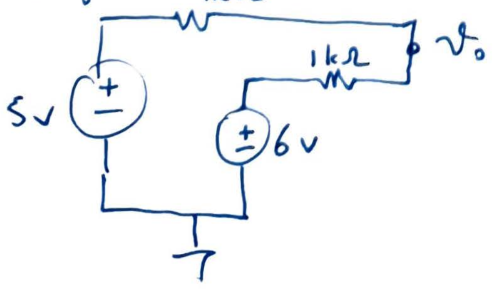
$$\frac{e_1 - 1}{3} + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + 2 = 0$$

$$-2 + \frac{e_2 - e_1}{2} + \frac{e_2}{5} - 1 = 0$$

$e_1 = 0.65\,V$

$e_2 = 4.75\,V \Rightarrow i = \frac{4.75}{5} = 0.95\,A$

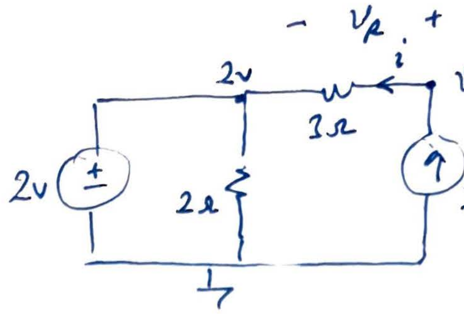
example



$$\frac{v_0 - 5}{1} + \frac{v_0 - 6}{1} = 0 \Rightarrow v_0 = 5.5\,V$$

N.T.

Ex.

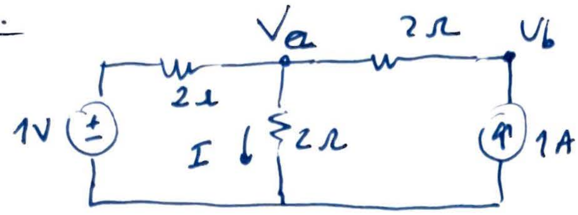


$v = ?$

KCL:

$$i = \frac{v_R}{3} = \frac{v-2}{3} = 3A \Rightarrow v = 11\ V$$

Ex.



$I = ?$

$$\frac{V_a - 1}{2} + \frac{V_a}{2} + \underbrace{\frac{V_a - V_b}{2}}_{-1} = 0$$

$$\frac{V_b - V_a}{2} = 1$$

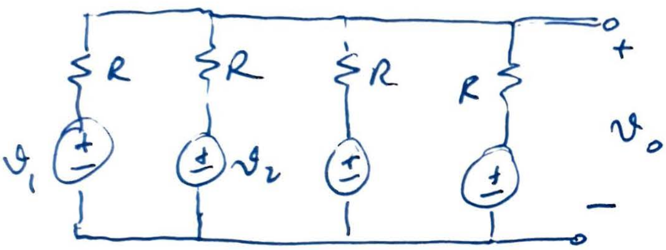
$$2V_a - 1 = 2$$

$$V_a = 1.5$$

$$I = 0.75\ A$$

Resistive adder circuit

Adding together many signals before amplifying them for example.

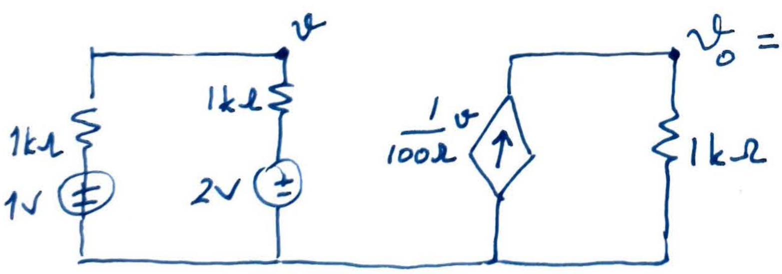


$$\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \cdots = 0$$

$$4v_o - (v_1 + v_2 + v_3 + v_4) = 0$$

$$v_o = \frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

Dependent Sources



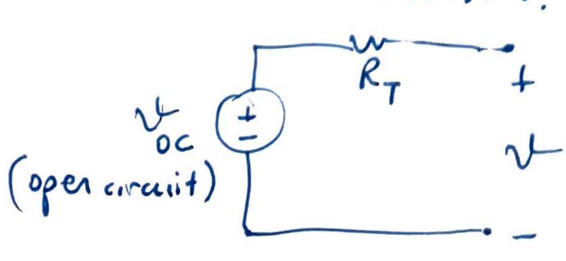
$v_o = ?$

$$v = \frac{(2V + 1V)}{2} = 1.5\ V$$

$$v_o = \frac{1.5}{100} \cdot 1000 = 15\ V$$

THEVENIN THEOREM

If the system is linear, any network can be represented by one voltage source and one resistor.

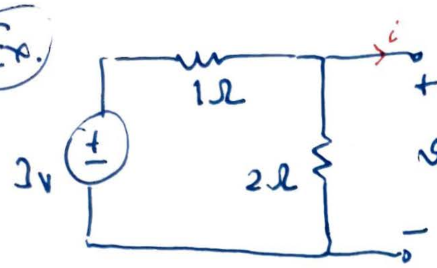


$$v_{oc} = v_t \Big|_{i_{test} = 0}$$

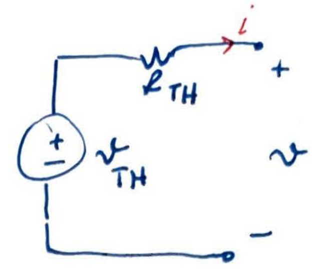
$$R_T = \frac{v_t}{i_{test}} \Big|_{\text{internal source} = 0}$$

* v_{TH} can be found by calculating open-circuit voltage at the designated terminal pair.
* R_{TH} can be found by calculating the resistance of the open-circuit network seen from the designated terminal pair.

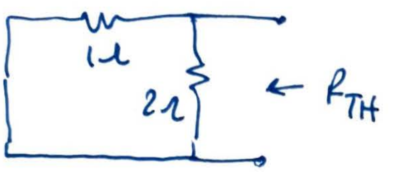
Ex.



$\equiv$

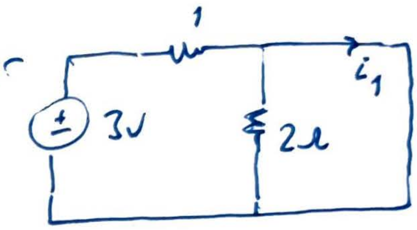


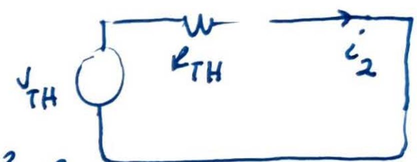
$$v_{TH} = 3V \cdot \frac{2\Omega}{1\Omega + 2\Omega} = 2V$$



$$R_{TH} = 1 \,||\, 2 = \frac{2}{3}\,\Omega$$

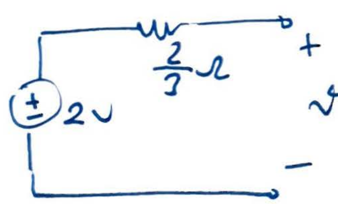
or



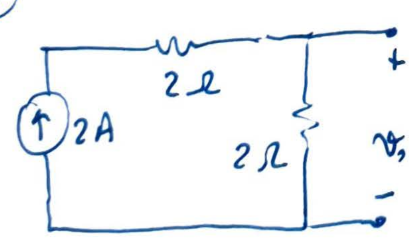


$$i_1 = i_2$$

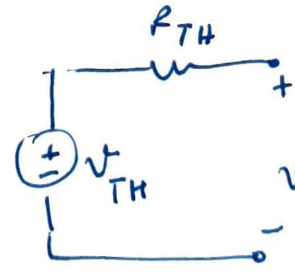
$$3 = \frac{v_{TH} = 2}{R_{TH}} \Rightarrow R_{TH} = \frac{2}{3}\,\Omega$$



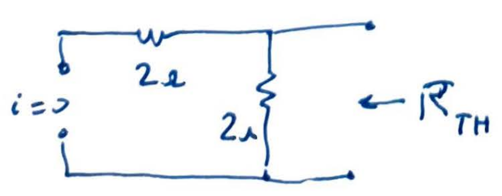
Ex



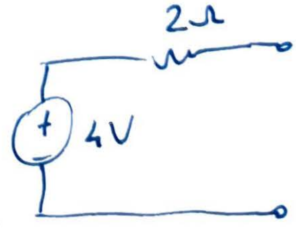
$\equiv$



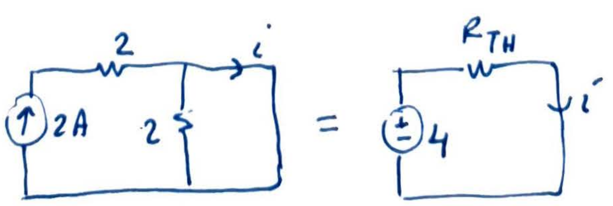
$$v_{TH} = 2A \cdot 2\Omega = 4V$$



$\Rightarrow R_{TH} = 2\,\Omega$ $\Rightarrow$



or short circuit:

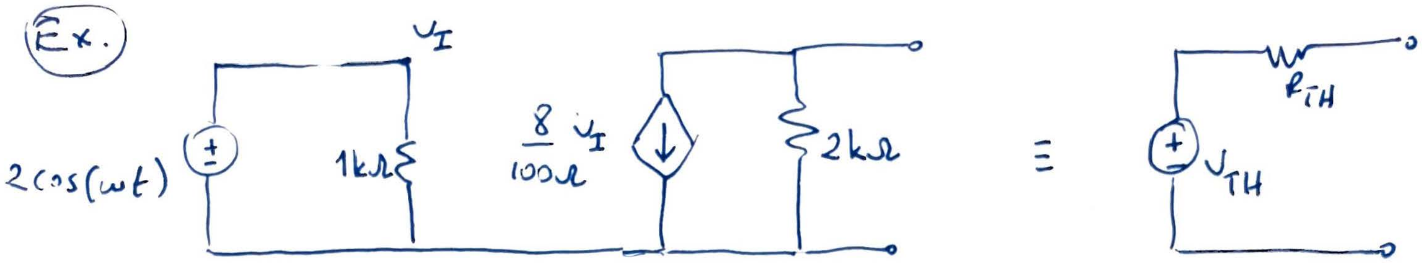


$i = 2$

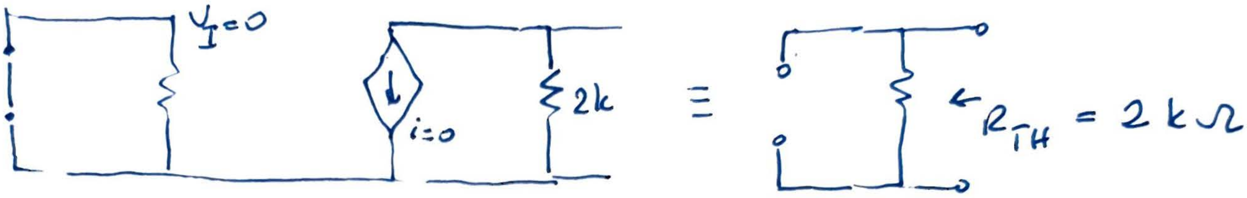
$$i = 2 = \frac{4}{R_{TH}} \Rightarrow R_{TH} = 2\,\Omega$$

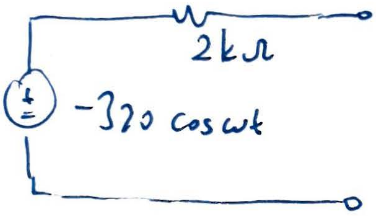
N.T.

Ex.



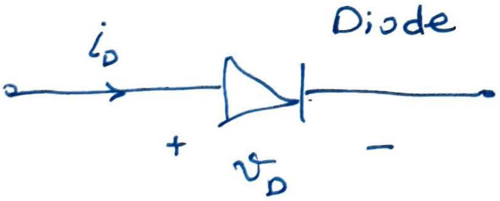
$$- \quad v_{TH} = \underbrace{2\cos(\omega t)}_{v_I} \cdot \frac{8}{100\Omega} \cdot 2k\Omega = 320 \cos(\omega t)$$

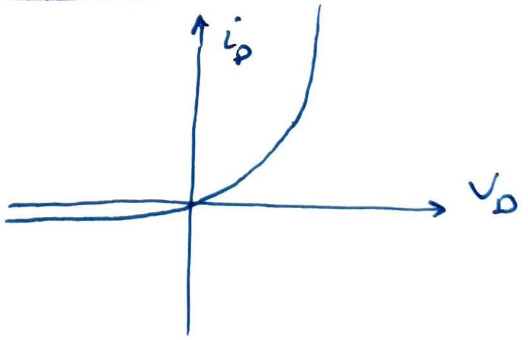




NON-LINEAR CIRCUITS

1





$$i_D = I_s \left(e^{v_D / v_{TH}} - 1 \right) \quad \text{where } v_{TH} = 0.025V, \; I_S = 10^{-12} A$$

Question: Determine the value of i_D for $v_D = 0.5$, 0.6 and 0.7 ?

$$i_D = 10^{-12} \cdot \left(e^{0.5/0.025} - 1 \right) = 0.49 \, mA$$

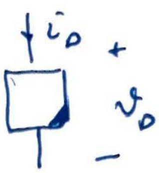
0.6 $\Rightarrow$ 26 mA

0.7 $\Rightarrow$ 1450 mA

dramatic increase

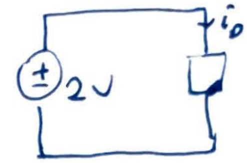
NON-LINEAR CIRCUITS

Another non-linear device: (Hypothetical)

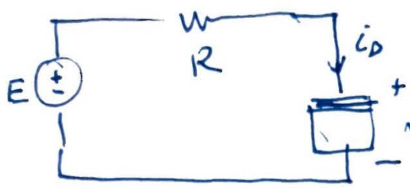


$$i_D = \begin{cases} 0.1\, v_D^2 & v_D \geq 0 \\ 0 & v_D < 0 \end{cases}$$

Example:



$i_D = 0.1 \cdot 2^2 = 0.4\ A$



$i_D = K v_D^2$, $v_D \geq 0$

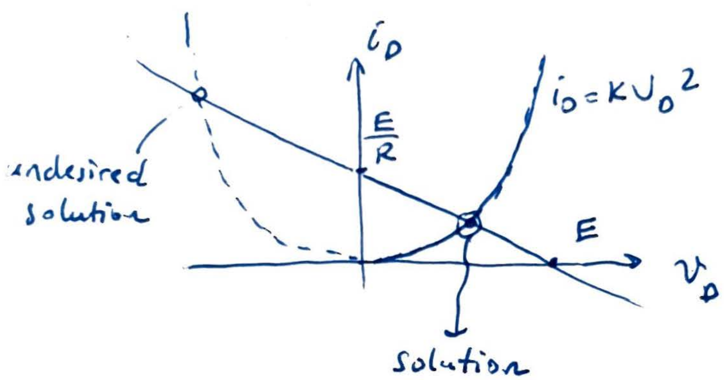
$i_D = 0$ otherwise

$$\frac{v_D - E}{R} + i_D = 0$$

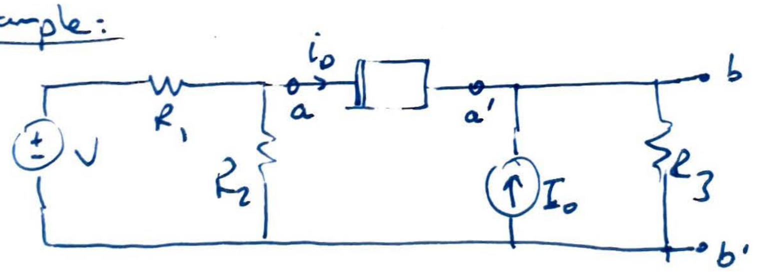
$$\frac{v_D - E}{R} + K v_D^2 = 0$$

$$RK v_D^2 + v_D - E = 0$$

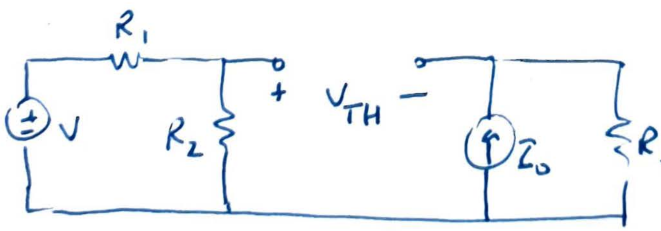
$$v_D = \frac{-1 + \sqrt{1 + 4RKE}}{2RK}$$

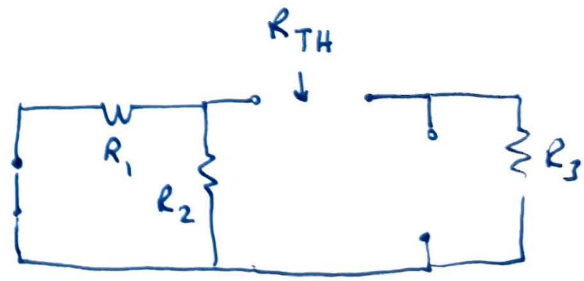


Example:



Perform Thevenin analysis:
(Terminals aa')





$$v_{TH} = v \frac{R_2}{R_1 + R_2} - I_o R_3$$

$$R_{TH} = (R_1 \| R_2) + R_3$$

Question: Can you find Thevenin equivalent at terminals b-b' ?