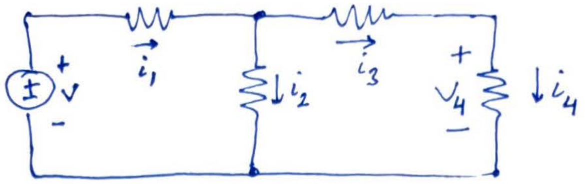
RESISTIVE NETWORKS

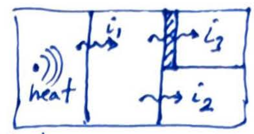
A simple electrical network (voltage source + resistors):



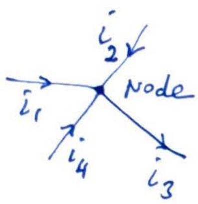
→ abstract representation of some real electrical network

or

→ model of a physical system (heat flow problem in a house)



Kirchhoff's Current Law (KCL)



Sum of currents flowing into a common point (node) is $\emptyset$.

$$\sum i = \emptyset = i_1 + i_2 - i_3 + i_4$$

Ex. $i_1 = +5A$, $i_2 = -3A$, $i_4 = +2A$, $\Rightarrow i_3 = ?$

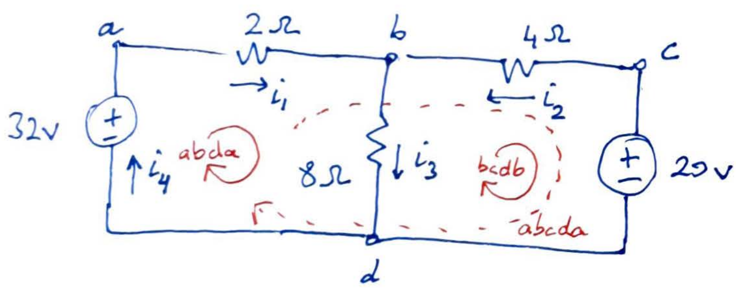
$$+5 - 3 - i_3 + 2 = 0$$

$$i_3 = +4\ A \checkmark$$

Kirchhoff's Voltage Law (KVL)

Sum of the voltages around a loop at any instant is zero.

$$\sum v = \emptyset.$$



① Apply KCL to node a:

$$\sum i_a = 0 = i_4 - i_1$$

$$i_1 = i_4$$

Current is the same at every point in a series circuit

$v = R \cdot i \Rightarrow v_{ab} = +2i_1$ (or $v_{ba} = -2i_1$

② $v_{bd} = +8i_3$ $\quad v_{db} = -8i_3$

$v_{cb} = +4i_2$ $\quad v_{bc} = -4i_2$)

③ Apply KCL to node b and d

$$\sum i_b = 0 = +i_1 + i_2 - i_3$$

$$\sum i_d = 0 = i_3 - i_4 - i_2$$

$$= i_3 - i_1 - i_2$$

④ Apply KVL to loop abda, bcdb:

$$\sum v = 0 = v_{ba} + v_{db} + v_{ad} \Rightarrow 0 = -2i_1 - 8i_3 + 32$$

$$\sum v = 0 = v_{cb} + v_{dc} + v_{bd} \Rightarrow 0 = +4i_2 - 20 + 8i_3$$

⑤ Apply KVL to loop abcda:

$$\sum v = 0 = v_{ba} + v_{cb} + v_{dc} + v_{ad}$$

$$\Downarrow$$

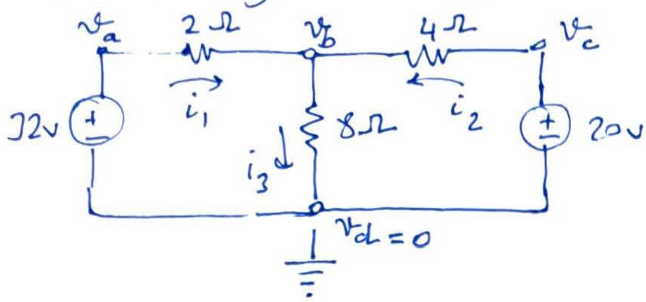
$$\left.\begin{aligned} i_1 + i_2 - i_3 &= 0 \\ +2i_1 \quad + 8i_3 &= 0 \\ +4i_2 + 8i_3 &= 0 \end{aligned}\right\} \xrightarrow{\text{Solve!}}$$

R.N.

$A \cdot x = b$, $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 8 \\ 0 & 4 & 8 \end{bmatrix}$, $x = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 32 \\ 20 \end{bmatrix}$ (2)

→ Solution by determinants, substitution method, Node-voltage (*)

Node-voltage method:



KCL to b: $\sum i_b = 0 = i_1 + i_2 - i_3$

$$i_1 = \frac{v_a - v_b}{2} = \frac{32 - v_b}{2}$$

$$i_2 = \frac{v_c - v_b}{4} = \frac{20 - v_b}{4}$$

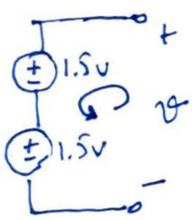
$$i_3 = \frac{v_b - 0}{8}$$

$$\Rightarrow \quad 0 = \frac{32 - v_b}{2} + \frac{20 - v_b}{4} - \frac{v_b}{8} \Rightarrow 128 - 4v_b + 40 - 2v_b - v_b = 0$$

$v_b = \frac{168}{7} = 24\,V$ $\quad i_3 = \frac{24}{8} = 3A$ $\quad i_1 = \frac{32-24}{2} = 4A$, $i_2 = \frac{20-24}{4} = -1A$

Examples:

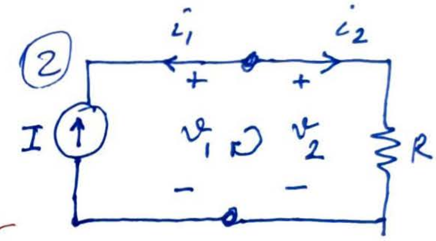
(1)



$+1.5v + 1.5v - v = 0$

$v = 3\,V$

(2)



$i_1 = -I$

$v_2 = R\, i_2$

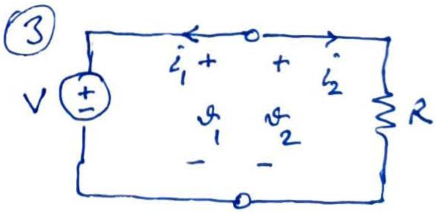
$i_1 + i_2 = 0$

$v_2 - v_1 = 0$

$\underline{v_1 = v_2 = RI}$

[energy conservation]

(3)



$v_1 = V$

$v_1 = v_2$

$i_1 + i_2 = 0$

$v_2 = R \cdot i_2$

$\underline{-i_1 = i_2 = \frac{V}{R} , \quad v_1 = v_2 = V}$

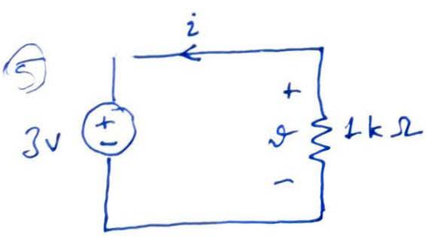
(4) Power into the current source:

$$i_1 v_1 = -RI^2$$

Power into the resistor

$$i_2 v_2 = RI^2 = R\frac{v^2}{R^2} = \frac{v^2}{R}$$

(5)



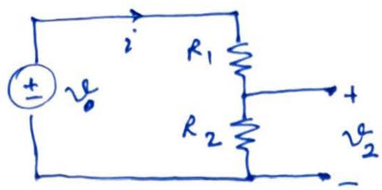
$i = 3mA$ is correct ?

energy dissipated by R is $i^2 \cdot R = 9\,mW$

energy into the source is $3v \cdot 3mA = 9\,mW$

it should be $-9\,mW$ ⇒ i must be $-\underline{3\,mA}$

Voltage Divider



$$i = \frac{v_0}{R_1+R_2}, \quad v_2 = R_2 \cdot i = R_2 \frac{v_0}{R_1+R_2}$$

$$v_2 = \frac{R_2}{R_1+R_2} \cdot v_0$$

In terms of conductance ($G = \frac{1}{R}$) : $v_2 = \frac{G_1}{G_1+G_2} \cdot v_0$

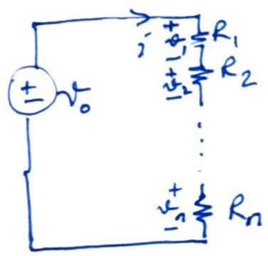
Ex. $v_0 = 10\,V$, $R_2 = 1\,k\Omega$. Choose R_1 such that v_2 is 10% of v_0.

$$v_2 = \frac{R_2}{R_1+R_2} \cdot v_0 \Rightarrow \frac{v_2}{v_0} = 0.1 = \frac{1\,k\Omega}{R_1+1\,k\Omega} \Rightarrow R_1 = 9\,k\Omega$$

Resistors in series :

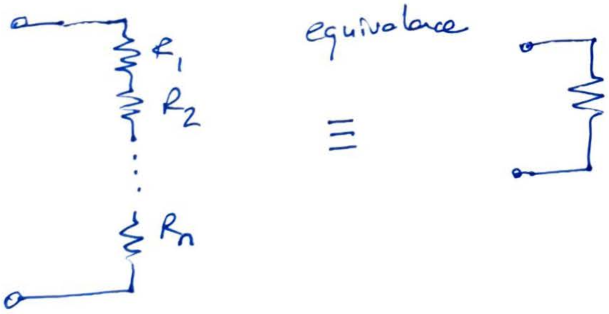
$$R_s = R_1 + R_2$$

N-Resistor Voltage Divider :



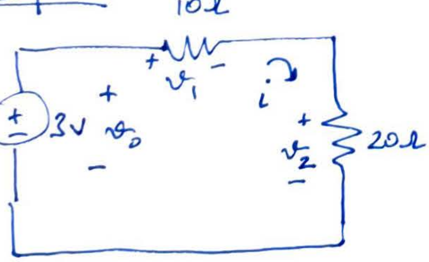
$$-v_0 + v_1 + \cdots + v_n = 0$$

$$i = \frac{v_0}{R_1+R_2+\cdots+R_n} \qquad v_n = \frac{R_n}{R_1+R_2+\cdots+R_n} \cdot v_0$$



$$R_S = R_1 + R_2 + \cdots + R_n$$

Example :

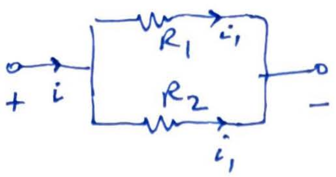


$$i = \frac{3}{30\Omega} = 0.1\,A$$

$$v_1 = R_1\, i = 10 \cdot 0.1 = 1\,V$$

$$v_2 = R_2\, i = 20 \cdot 0.1 = 2\,V$$

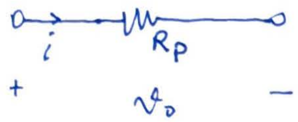
Resistors in Parallel:



$i = i_1 + i_2 \qquad i_1 = \dfrac{v_0}{R_1} \qquad i_2 = \dfrac{v_0}{R_2}$

v_0

||| equiv.



$$i = \frac{v_0}{R_p} = \frac{v_0}{R_1} + \frac{v_0}{R_2} \Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{or} \quad R_p = \frac{R_1 R_2}{R_1 + R_2}$$

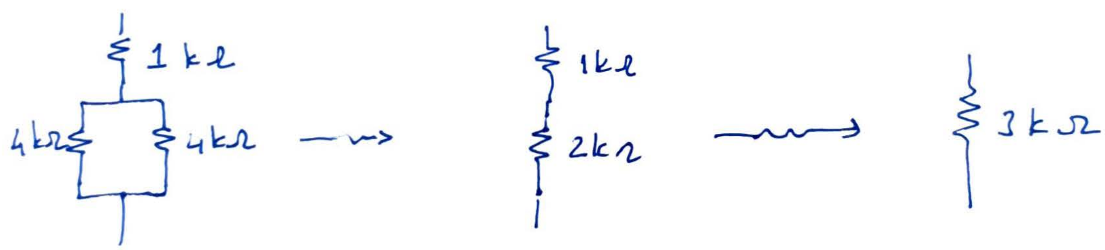
we use sometimes this notation:

$$R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

for n resistor case: $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}$

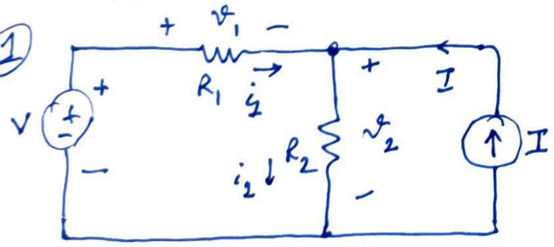
for n resistor equal ($R_i = R \ \forall i$) : $R_p = \dfrac{R}{n}$

Equivalent resistors:



More complex example:

(1)



$I + i_1 = i_2 \qquad i_2 = \dfrac{v_2}{R_2} \qquad i_1 = \dfrac{V - v_2}{R_1}$

$$\underset{(R_1R_2)}{\frac{I}{1}} + \underset{(R_2)}{\frac{V - v_2}{R_1}} = \underset{(R_1)}{\frac{v_2}{R_2}} \Rightarrow$$

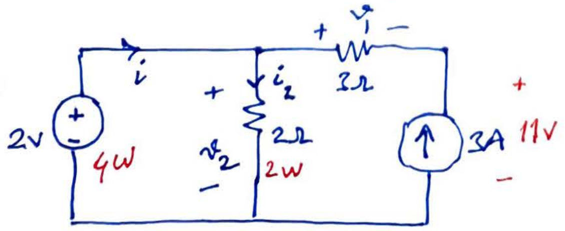
$$I R_1 R_2 + V R_2 - v_2 R_2 = v_2 R_1$$

$$I R_1 R_2 + V R_2 = v_2 (R_1 + R_2) \Rightarrow v_2 = \frac{I R_1 R_2 + V R_2}{R_1 + R_2}$$

$$v_2 = \frac{R_1 R_2}{R_1 + R_2} I + \frac{R_2}{R_1 + R_2} V$$

$i^2 \cdot R = 9 \cdot 3 = 27W$

(2)



$- i_2 + i + 3 = 0 \Rightarrow i = -2A$

$i_2 = \dfrac{2}{2\Omega} = 1A$

Power dis. in resistors: 29W

Power into sources:

$-11V \cdot 3A + 2V \cdot 1A = -29 \text{ W}$

Show the energy conservation!