CENG504 Optimization Methods Term Project

Traveling Salesman Problem

Introduction

The traveling salesman problem (TSP) is one which has commanded much attention of mathematicians and computer scientists specifically because it is so easy to describe and so difficult to solve. The problem can simply be stated as: if a traveling salesman wishes to visit exactly once each of a list of m cities (where the cost of traveling from city i to city j is c_{ij}) and then return to the home city, what is the least costly route the traveling salesman can take?

The importance of the TSP is that it is representative of a larger class of problems known as combinatorial optimization problems. The TSP problem belongs in the class of combinatorial optimization problems known as NP-complete. Specifically, if one can find an efficient algorithm (i.e., an algorithm that will be guaranteed to find the optimal solution in a polynomial number of steps) for the traveling salesman problem, then efficient algorithms could be found for all other problems in the NP-complete class. To date, however, no one has found a polynomial-time algorithm for the TSP.

Formulation

Each city is denoted by a point (or node) and lines are drawn connecting every two nodes (called arcs or edges). Associated with every line is a distance (or cost). When the salesman can get from every city to every other city directly, then the graph is said to be complete. A round-trip of the cities corresponds to some subset of the lines, and is called a tour or a Hamiltonian cycle in graph theory. The length of a tour is the sum of the lengths of the lines in the round-trip. Let us assume that there are m cities. We introduce x_{ij} such that

$$x_{ij} = \begin{cases} 1 & \text{if the edge } \ i \to j \ \text{ is in the tour} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The objective function is:

$$\min \sum_{j=1}^{m} \sum_{i=1}^{m} x_{ij} c_{ij} \tag{2}$$

We organize the constraints as follows. Notice that every node of the graph must have exactly one edge pointing towards it and one pointing away from it. Therefore we have

$$\sum_{j=1}^{m} x_{ij} = 1 \quad \forall i \in \{1, \ldots, m\} \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad \forall j \in \{1, \ldots, m\} \tag{4}$$

These constraints alone are not enough since this formulation would allow "subtour", that is, it would allow disjoint loops to occur. For this reason, a proper formulation of the traveling salesman problem must remove these subtours from consideration by the addition of "subtour elimination" constraints:

$$\sum_{i \in K} \sum_{j \in K} x_{ij} = \mid K \mid -1 \quad \forall K \subset \{1, \ldots, m\} \tag{5}$$

$$x_{ij} = 0 \text{ or } 1 \quad \forall i, j \tag{6}$$

where K is subset of cities.

You are supposed to solve the problem using one of the following optimization techniques:

1. Linear Programming (Melike Kaptan)
2. Simulated Annealing (Deniz Kavzak)
3. Stochastic Gradient Descent (Arzum Karatas)
4. Simulated Annealing (Orhan Bayraktar)
5. Genetic Algorithm (Onur Temizkan)
6. Particle Swarm Optimization (David Thera)
7. Bee Colony Optimization (Dilek Ozturk)
8. Tabu Search (Ozan Polatbilek)
9. River Formation Dynamics (Ceren Atik)
10. Ant Colony Optimization (Damla Yasar)

Please prepare a presentation for maximum 20-25 minutes, introducing the problem, similar solutions using your method in the literature if any, and your implementation with your results or just your ideas.

You are also expected to present your implementation in a research paper format and submit it to the lecturer by the deadline of June 15th, 2018.

To test your implementation, you can use the following TSP problem:

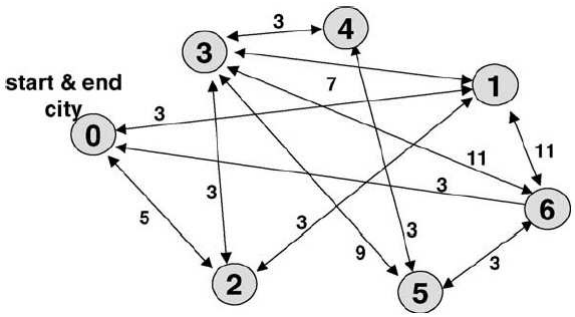

Figure 1: TSP with 7 cities

Grading

Oral Presentation: 25%

Research Report: 75%

The grading details of the report is as follows:

- General Overview: 10 pts
- Research Paper Format and Syntax: 20 pts
- Quality of the Literature Review/Related Work: 20 pts
- Implementation: 25 pts
  - Originality of your method: 5 pts
  - Problem Formulation (without implementation): 10 pts
  - Fully implementation with the results: 10 pts